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Dual synchronization based on two different chaotic systems: Lorenz systems and Rössler systems ☆

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Abstract

In this paper, we improve and extend the works of Liu and Davids [Dual synchronization of chaos, Phys. Rev. E 61 (2000) 2176–2179] which only introduce the dual synchronization of 1-D discrete chaotic systems. The dual synchronization of two different 3-D continuous chaotic systems, Lorenz systems and Rössler systems, is discussed. And a sufficient condition of dual synchronization about the two different chaotic systems is obtained. Theories and numerical simulations show the possibility of dual synchronization and the effectiveness of the method.

MSC: 34C28; 37D45; 37L45

Keywords: Dual synchronization; Lyapunov stabilization theory; Secure communication

1. Introduction

The applications of nonlinear chaotic oscillators in communications were proposed about two decades ago. Synchronization [7] plays an important role in chaotic communications because it offers a potential advantage over noncoherent detection in terms of noise performance and data when the basis functions are recovered from noisy distorted received signals [2,3,11]. In recent years multiuser chaotic communications have become a hot topic [5,8].

For linear communication systems, there are a number of standard ways, such as frequency division multiplexing and time division multiplexing, to increase the information capacity of the channel by sending multiple signals over one channel. For chaotic communication systems, it would also be of great interest to exploit the property of multiplexing chaotic signals in one communication channel. In 1996, multiplexing chaos using synchronization was investigated in a simple map and an electronic circuit model by Tsimring and Sushchik for the first time [9]. Then in 2000 Liu and Davids raised the concept of "dual synchronization", which refers to using a scale signal to simultaneously synchronize two different pairs of chaotic oscillator (two masters and two slaves) [4]. The authors investigated multiplexing chaos for

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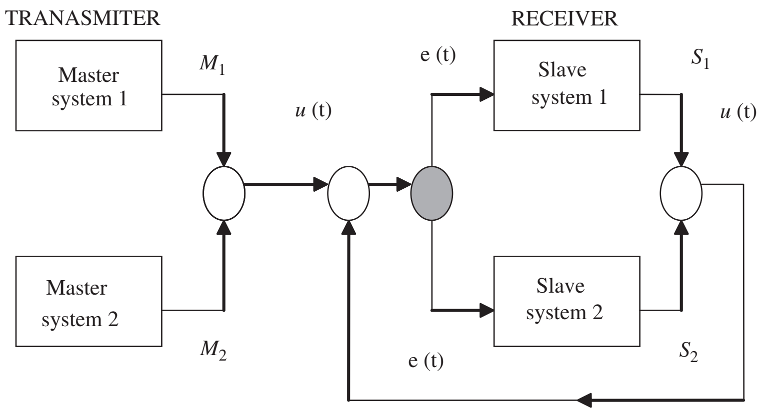

Fig. 1. Error signals between the pair of Lorenz systems.

two different pairs of chaotic maps and delay-differential equations. In [1,4,9,10] they demonstrated that numerically multiplexing chaotic signal generated by two different chaotic systems, which are the same dynamic systems with slightly different parameters or the two totally different dynamic systems (one is a Colpitts oscillator, the other is a Chua's circuit), is synchronized using dual synchronization. However, the previous works just give relative numerical simulation. The works of Liu and Davids [4] introduce the dual synchronization of 1-D discrete chaotic systems. We further improve and extend their works. The dual synchronization based on two different 3-D continuous chaotic systems, Lorenz systems and Rössler systems, is discussed. And the sufficient condition of dual synchronization between two different chaotic systems is obtained too. Furthermore, we demonstrate the possibility of dual synchronization by means of Lyapunov stabilization theory which is used to synchronize two different chaotic systems in [6]. Numerical simulations are shown for demonstration.

The dual synchronization configuration of one Lorenz system pair and one Rössler system pair is plotted in Fig. 1.

2. Dual synchronization

We define the Master systems and Slave systems as follows:

Master systems:

$$M_1 : \begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - y - xz, \\ \dot{z} = xy - bz. \end{cases} \tag{1}$$

$$M_2 : \begin{cases} \dot{l} = -(n + l), \\ \dot{m} = l + \alpha m, \\ \dot{n} = nl - \gamma n + \beta. \end{cases} \tag{2}$$

Slave systems:

$$S_1 : \begin{cases} \dot{x}_1 = a(y_1 - x_1) + w_1 e, \\ \dot{y}_1 = cx_1 - y_1 - x_1 z_1 + w_2 e, \\ \dot{z}_1 = x_1 y_1 - bz_1 + w_3 e, \end{cases} \tag{3}$$

$$S_2 : \begin{cases} \dot{l}_1 = -(n_1 + l_1) + w_4 e, \\ \dot{m}_1 = l_1 + \alpha m_1 + w_5 e, \\ \dot{n}_1 = n_1 l_1 - \gamma n_1 + \beta + w_6 e, \end{cases} \tag{4}$$

where $a, b, c, \alpha, \beta, \gamma$ are system parameters. $w_i\,(i = 1, 2, \ldots, 6)$ are feedback gains. e is defined as (5).

The dual synchronization state is defined as $M_1 = S_1$, $M_2 = S_2$. Clearly, the solution of such dual synchronization state exists. For example, if the initial state is chosen as $M_1(0) = S_1(0)$, $M_2(0) = S_2(0)$, the error signals are zero and remain zero. Then the oscillations remain identical. We next show that the dual synchronization state can also be an attracting solution by Lyapunov stabilization theory with respect to the synchronization state $M_1 = S_1$, $M_2 = S_2$.

Lemma 1. *Suppose that $A \in R^{n\times n}$ is a negative definite matrix, there exists a neighborhood U containing $x = 0$ for given $b > 0$, such that $x^{\mathrm{T}}Ax + b\|x\|^3 < 0$, for all $x \in U \subset R^n$, $x \neq 0$, $\|\cdot\|$ is Euclidean norm.*

Proof. Since $A \in R^{n\times n}$ is a negative definite matrix, there exists an orthogonal matrix B, such that $B^{\mathrm{T}}AB = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) = P$, where $\lambda_1 \leqslant \lambda_2 \leqslant \cdots \leqslant \lambda_n < 0$ are eigenvalues of A.

Define a transitions $y = B^{\mathrm{T}}x$, then $x^{\mathrm{T}}Ax = y^{\mathrm{T}}Py$ and $\|x\| = \|y\|$. Since $x \neq 0$, $y \neq 0$, that is $\|y\| \neq 0$, $x^{\mathrm{T}}Ax + b\|x\|^3 = y^{\mathrm{T}}Py + b\|y\|^3 \leqslant \lambda_n\|y\|^2 + b\|y\|^3 = \|y\|^2(\lambda_n + b\|y\|)$.

Assume $b > 0$, because $\lambda_n < 0$, there exists a small enough neighborhood $U(y)$ which contains $y = 0$, such that $\lambda_n + b\|y\| < 0$.

Consequently, there $U = U(x)$ is containing $x = 0$, such that $x^{\mathrm{T}}Ax + b\|x\|^3 \leqslant \|y\|^2(\lambda_n + b\|y\|) < 0$, for all $x \in U \subset R^n$, $x \neq 0$. □

Define $M_1 = (x, y, z)^{\mathrm{T}}$, $M_2 = (l, m, n)^{\mathrm{T}}$, $S_1 = (x_1, y_1, z_1)^{\mathrm{T}}$, $S_2 = (l_1, m_1, n_1)^{\mathrm{T}}$. Let $u = AM_1 + BM_2$ denote the linear coupling of M_1 and M_2. And $u' = AS_1 + BS_2$ denote the linear coupling of S_1 and S_2. $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3)$ are couple parameters.The feedback gain function $w_i (i = 1, 2, \ldots, 6)$ is 3-D column vector:

$$e = u' - u = a_1e_1 + a_2e_2 + a_3e_3 + b_1e_4 + b_2e_5 + b_3e_6, \tag{5}$$

where e is a scalar signal.

$$e_1 = x_1 - x, \quad e_2 = y_1 - y, \quad e_3 = z_1 - z, \quad e_4 = l_1 - l, \quad e_5 = m_1 - m, \quad e_6 = n_1 - n.$$

The following is the error dynamics:

$$\begin{cases} \dot{e_1} = ae_2 - ae_1 + w_1e, \\ \dot{e_2} = ce_1 - e_2 - e_1e_3 - e_1z - xe_3 + w_2e, \\ \dot{e_3} = e_1e_2 + e_1y + xe_2 - be_3 + w_3e, \\ \dot{e_4} = -e_6 - e_4 + w_4e, \\ \dot{e_5} = e_4 + \alpha e_5 + w_5e, \\ \dot{e_6} = e_6e_4 + e_6l + ne_4 - \gamma e_6 + w_6e. \end{cases} \tag{6}$$

Construct a Lyapunov function of the form: $V = \frac{1}{2}\eta^{\mathrm{T}}\eta + \frac{1}{2}\sum(w_i - w_i^*)^2$, where w_i^* is a constant.

If $\dot{w}_i = -e_ie (i = 1, 2, \ldots, 6)$, $|y| < M_y$, $|z| < M_z$, then its time derivative along system (6) is

$$\begin{aligned} \dot{V} &= \frac{1}{2}(\dot{\eta}^{\mathrm{T}}\eta + \eta^{\mathrm{T}}\dot{\eta}) + \sum(w_i - w_i^*)\dot{w}_i \\ &= \dot{e}_1e_1 + \dot{e}_2e_2 + \dot{e}_3e_3 + \dot{e}_4e_4 + \dot{e}_5e_5 + \dot{e}_6e_6 + \sum(w_i - w_i^*)\dot{w}_i \\ &= (w_1^*a_1 - a)e_1^2 + (a + c - z + w_1^*a_2 + w_2^*a_1)e_1e_2 + (w_1^*a_3 + w_3^*a_1 + y)e_1e_3 \\ &\quad + (w_1^*b_1 + w_4^*a_1)e_1e_4 + (w_1^*b_2 + w_5^*a_1)e_1e_5 + (w_1^*b_3 + w_6^*a_1)e_1e_6 \\ &\quad + (w_2^*a_2 - 1)e_2^2 + (w_2^*a_3 + w_3^*a_2)e_2e_3 + (w_2^*b_1 + w_4^*a_2)e_2e_4 + (w_2^*b_2 + w_5^*a_2)e_2e_5 \\ &\quad + (w_2^*b_3 + w_6^*a_2)e_2e_6 + (w_3^*a_3 - b)e_3^2 + (w_3^*b_1 + w_4^*a_3)e_3e_4 + (w_3^*b_2 + w_5^*a_3)e_3e_5 \\ &\quad + (w_3^*b_3 + w_6^*a_3)e_3e_6 + (w_4^*b_1 - 1)e_4^2 + (w_4^*b_2 + w_5^*b_1 + 1)e_4e_5 + (w_4^*b_3 + w_6^*b_1 - 1 + n)e_4e_6 \\ &\quad + (w_5^*b_2 - \alpha)e_5^2 + (w_5^*b_3 + w_6^*b_2)e_5e_6 + (w_6^* + l - \gamma)e_6^2 + e_4e_6^2 \\ &\leqslant -\eta^{\mathrm{T}}P\eta + |e_4|e_6^2, \end{aligned}$$

where $\eta = (|e_1|, |e_2|, |e_3|, |e_4|, |e_5|, |e_6|)^T$. P is a real symmetric matrix. And P is determined by parameters A, B and w_i^*.

Obviously, P should be positive definite in order to obtain the locally asymptotically stability of the original solution of errors system. P is positive definite if and only if

$$\Delta_i > 0, \quad i = 1, 2, \ldots, 6, \tag{7}$$

where Δ_i represents the ith order sequential subdeterminant of matrix. That is, we should choose the appropriate parameters.

With the aid of numeric simulation, we can obtain the scope of symbolic variables on the condition that P is positive definite. By Lemma 1, we obtain $\dot{V} < 0$. Based on Lyapunov stabilization theory, we have $|e_i| \to 0$ as $t \to \infty$ for $i = 1, 2, \ldots, 6$. Then, we realize the dual synchronization between Lorenz systems and Rössler systems.

Proposition 1. *The two drive systems* M_1, M_2 *and the two controlled systems* S_1, S_2 *can be dual synchronization, if* $\dot{w}_i = -e_i e (i = 1, 2, \ldots, 6)$ *and the coupled parameters satisfy* (7).

3. Numerical simulations

Fourth-order Runge–Kutta method is used to solve the systems of differential equations in all numerical simulations. Let $a = 10$, $b = \frac{8}{3}$, $c = 28$ and $\alpha = \beta = 0.2$, $\gamma = 5.7$, which are chaotic systems for M_1 and M_2. And select the initial values of systems as follows:

$$M_1 : (x, y, z)|_{t=0} = (0.2, 0.6, 1), \quad S_1 : (l, m, n)|_{t=0} = (0.5, 1, 1.5),$$

$$M_2 : (x, y, z)|_{t=0} = (0.1, 0.2, 1.1), \quad S_2 : (l, m, n)|_{t=0} = (2.5, 2, 2.5).$$

In addition, the coupled parameters are valued as $A = (2, 2, -1)$, $B = (3, -2, 1)$, for which condition (7) is satisfied. Figs. 2 and 3 display the errors of the dual synchronization.

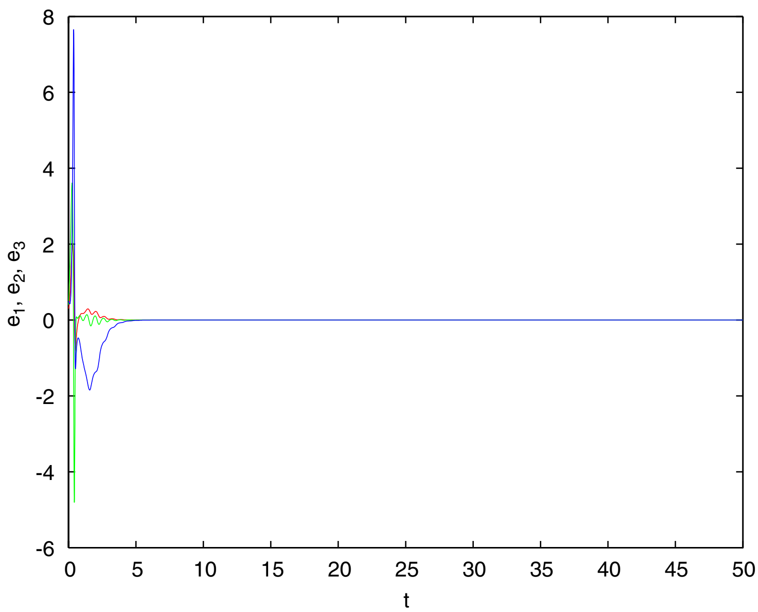

Fig. 2. Error signals between the pair of Lorenz systems.

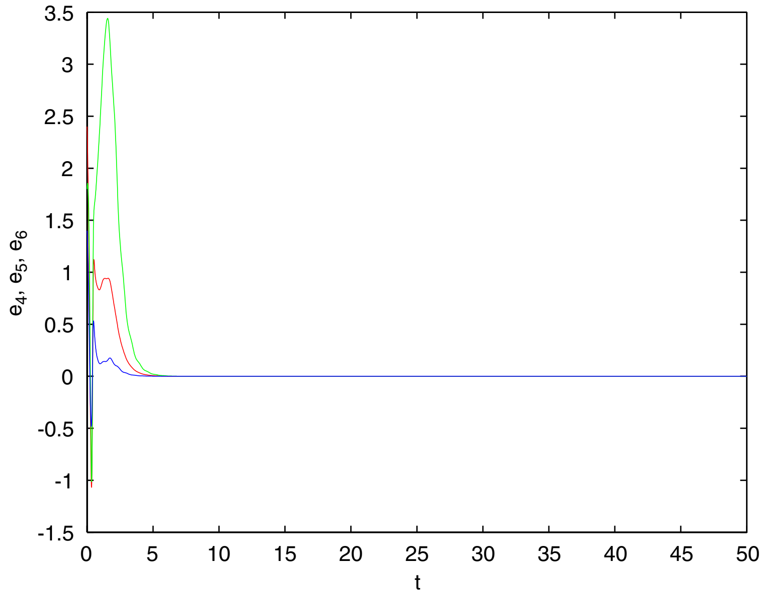

Fig. 3. Error signals between the pair of Rössler systems.

4. Conclusions

In this paper, we introduce dual synchronization between Lorenz system and Rössler system, and propose the sufficient condition of dual synchronization about two different chaotic systems. In addition, we demonstrate the possibility of dual synchronization in theory. Numerical simulations show the possibility of dual synchronization and the effectiveness of the method.

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